On the theory and practice of water regulation

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Abstract

We study water regulation for a schematic water economy representing a wide range of real world situations. A water policy has inter- and intra-temporal components. The first determines the limits on extractions from the naturally replenished sources, given the stochastic nature of recharge processes associated with uncertain precipitation. The intra-temporal regulation is concerned mainly with the allocating of the extracted and produced water among the end-users. The prices that implement the optimal intra-temporal allocation are derived. Regulation issues associated with cost recovery and asymmetric information are discussed.

Keywords: scarcity, pricing, optimal allocation, water economy,

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1 Introduction

Population growth and rising living standards have led to a rapid increase in the demand for water. As the quantity of renewable fresh water available for use in any particular location is on average constant and water conveyance is an expensive operation, water has become scarce in many parts of the world. Adding the prevalence of deteriorating water quality and the increased awareness for water-related environmental and social problems helps to understand why water regulation has become a critical policy challenge. The goals of a water policy entail efficient use of the existing sources and a balanced planning and development of new sources. As most economically viable, natural sources have already been developed, prospects for augmented water supply increasingly rely on secondary sources such as recycled and desalinated water.

This work presents the basic principles of regulating a water economy. Water economies vary with respect to physical and social properties, and a successful policy must be tailored to the particular conditions of the case under consideration. Our focus here is on those principles shared by many water policies, in spite of the idiosyncrasies of the water economy to which they are applied.

The water economy is described in Section 2. In Section 3 we define feasible and optimal water policies. A water policy has inter- and intra-temporal components. The first determines the limits on extractions from the naturally replenished sources, given the fluctuating nature of recharge processes due to stochastic precipitation. The intra-temporal regulation is concerned with the allocation of the extracted and produced water among the end-users and allocation of the supply infrastructure (capital).
4 specifies the water prices that implement the intra-temporal allocation. It highlights an interesting finding that the optimal water price of a particular user should not be affected by the cost of water that will never be demanded by that user (e.g., the price of irrigation water should not be affected by the cost of desalination). Section 5 discusses regulation issues associated with cost recovery and asymmetric information. Section 6 remarks on a variety of frequently encountered issues that lead to departure from the optimal pricing of Section 4. The appendix contains technical details and derivations.

We note at the outset that this effort does not pertain to survey the wide range of water regulation issues and no attempt is made to cover the huge literature on this topic. Our aim is to lay out the important principles of water regulation in a concise and coherent fashion and in a way that can be used in actual implementation.

2 The water economy

A water economy consists of (i) the physical resource base (precipitation, rivers, lakes, aquifers), (ii) consumers and users (irrigators, households, industry), (iii) suppliers and the associated infrastructure (extraction-conveyance-treatment infrastructure), and (iv) regulatory and institutional infrastructure (water laws and property rights, prices and quotas, water institutions). We begin with a schematic description of these components.

2.1 Water resources

There are $M$ (possibly interconnected) naturally replenished water sources (rivers, lakes, reservoirs, aquifers) whose stocks at time $t$ are represented by
\( Q_t = (Q_1^t, Q_2^t, ..., Q_M^t) \). The water stocks evolve in time according to
\[
Q_{t+1}^m = Q_t^m + R^m(Q_t) + x_t^m - g_t^m, \quad m = 1, 2, ..., M, \tag{2.1}
\]
where \( R^m(\cdot) \) represents deterministic recharge, \( x_t^m \) is stochastic recharge and \( g_t^m \) is the rate of extraction from source \( m \).

Recharge at time \( t \) emanates from current precipitation and from subsurface flows. The latter depends on current and past precipitation. Precipitation may vary spatially across the basin. Accordingly, we divide the basin into \( N \geq 1 \) subregions and denote by \( w_t = (w_1^t, w_2^t, ..., w_N^t) \) the precipitation in the \( N \) subregions during period \( t \). The \( w_t \)'s are i.i.d. draws from an \( N \)-dimensional distribution \( F_w \) defined over a nonnegative support.

Current and past precipitations generate the stochastic recharge \( x_t = (x_1^t, x_2^t, ..., x_M^t) \) according to
\[
x_{t+1} = w_{t+1} \Lambda + x_t \Gamma, \tag{2.2}
\]
where \( \Lambda \) and \( \Gamma \) are, respectively, \( N \times M \) and \( M \times M \) matrices of (known) coefficients. The \( m \)'th column of \( \Lambda \) represents the immediate effect of precipitation on the \( m \)'th stock recharge, while the \( m \)'th column of \( \Gamma \) represent the (diminishing) effects of past precipitation. In view of (2.2), the water stocks evolution (2.1) can be rendered as
\[
Q_{t+1} = Q_t + R(Q_t) + x_t \Gamma + w_{t+1} \Lambda - g_t, \tag{2.3}
\]
where \( R(Q) = (R^1(Q), R^2(Q), ..., R^M(Q)) \) and \( g_t = (g_t^1, g_t^2, ..., g_t^M) \).

Two types of produced sources may also be available: desalinated water (of brackish sources or seawater) and recycled (treated sewage) water. We refer to desalination as source \( M + h, \ h = 1, 2, ..., H \), where \( H \) is the number of desalination plants.
Recycled water has two distinctive features that separate it from the other sources. First, exogenous (health and environmental) regulations often require treating sewage water, disregarding whether it will later be reused. Second, the same regulations often forbid mixing treated sewage water with water from the other sources, implying that reusing the treated water requires separate conveyance and distribution systems. These properties affect the pricing of recycled water, discussed below.

2.2 Consumers and users

The basin contains $S$ private sectors (urban, agriculture, industry) and a few public sectors (parks, estuaries, wilderness areas) scattered spatially in $L$ locations (districts, regions, municipalities). We consider a single public sector, called the environment (e.g., instream water), indexed $S + 1$.\footnote{Water allocated to the environment has features of a public good, hence the analysis of this sector differs from that of the $S$ private sectors.} The inverse water demand for sector $s = 1, 2, ..., S$, in location $l = 1, 2, ..., L$, is denoted $D^{sl}(\cdot)$: when the water price ($\$ \text{ per m}^3$, say) is $D^{sl}(q)$, sector $s$ in location $l$ demands the water quantity $q$. We assume stationary water demands; extensions needed to account for non-stationary effects (e.g., economic and demographic growth) will be discussed in the concluding section.

2.2.1 Agricultural (irrigation) demand

The number of agricultural sectors depends on the level of aggregation and may contain, for example, orchards, vegetables, fiber (cotton), cereals, other field crops and livestock. Agricultural sector $s$ in location $l$ has $J$ activities (crops), indexed $j = 1, 2, ..., J$. Let $y_j(q)$ denote crop $j$’s water-yield value
function (not including the water cost).\footnote{These functions are defined as follows: Let \( \tilde{y}_j(q, b, z) \) denote crop \( j \) production function, where \( q \) is water input, \( b \) is a vector of fixed inputs (e.g., land and family labor) and \( z \) is a vector of purchased inputs (labor, fertilizers, pesticides, machinery) with price vector \( r \). Then, \( y_j(q) = \max_z \{ p_j \tilde{y}(q, z, b) - rz \} \) s.t. \( b \leq \bar{b} \), where the output price \( p_j \), the fixed inputs constraint \( b \) and the input prices \( r \) are suppressed as arguments.} The corresponding inverse demand for irrigation water is given by \( y'_j(\cdot) \equiv \partial y_j(\cdot)/\partial q \). To see this note that when the price of water is \( p_w \), profit is \( y_j(q) - p_wq \) and the water input that maximizes profit satisfies \( y'_j(q) = p_w \). Thus, the water demand at that price is \( y^{-1}_j(p_w) \). Typically, \( y_j(\cdot) \) is increasing and strictly concave, so that \( y'_j(\cdot) \) is decreasing and its inverse exists. The water demand of agricultural sector \( s \) in location \( l \) is \( q_{sl}(p_w) = \sum_j y^{-1}_j(p_w) \) and the corresponding inverse demand is \( D_{sl}(\cdot) = q_{sl}^{-1}(\cdot) \). The diminishing marginal productivity of water implies that \( D_{sl}(\cdot) \) is decreasing (see details in Tsur et al. 2004, Tsur 2005).

### 2.2.2 Industrial demand

Industrial sectors contain non-agricultural production activities that use water as an input of production. As above, the number of industrial sectors depends on the level of aggregation and the sectors are defined according to the role and use of water in the production process. The inverse water demand of industrial sector \( s \) in location \( l \), \( D_{sl}(\cdot) \), is derived in the same way as the agricultural water demand, with industrial activities instead of agricultural activities (see Renzetti 2002a, for a detailed analysis).

### 2.2.3 Residential demand

The utility of household \( i \) depends on the per-capita consumption of water \( \tilde{q} \) and other goods \( \tilde{z} \). The (per-member) demands for \( \tilde{q} \) and \( \tilde{z} \) are the
outcome of
\[
v_i(p_w, p_z, y_i, n_i) = \max_{\tilde{q}, \tilde{z}} u_i(\tilde{q}, \tilde{z}) \quad s.t. \quad (p_w \tilde{q} + p_z \tilde{z}) n_i \leq y_i,
\]
where \(y_i\) is the household’s income, \(n_i\) is the household’s size (number of members) and \((p_w, p_z)\) are the prices of \((\tilde{q}, \tilde{z})\). Household \(i\)’s (per capita) water demand is denoted \(\tilde{q}_i(p_w, p_z, y_i, n_i)\) and the residential water demand in location \(l\) is (retaining only the water price argument)
\[
q_{sl}(p_w) = \sum_{i \in \text{location } l} n_i \tilde{q}_i(p_w, p_z, y_i, n_i)
\]
and the corresponding inverse water demand is \(D_{sl}(\cdot) = q_{sl}^{-1}(\cdot)\). The residential sector includes water use for human needs (including water consumed in service, public and commercial institutions) and private gardening (water use in public urban parks is included in the environmental sector, discussed below, due to its public-good feature). With some added complication, it is possible to consider private gardens as an additional residential sector (detailed accounts can be found in Baumann et al. 1998, Renzetti 2002b).

2.2.4 Environmental demand

Environmental sectors include irrigation water of public urban parks and instream water in wilderness areas and estuaries. They differ from the sectors discussed above due to their public good features. We briefly outline how to incorporate environmental water, assuming for simplicity a single environmental sector indicated as sector \(E\) or \(S+1\) interchangeably. Let \(q^{E_l} \equiv q^{S+1_l}\) represent allocation of environmental water in location \(l\). Household’s \(i\) demand for \(q^{E_l}\) is measured in terms of the household’s willingness to pay (WTP) to preserve \(q^{E_l}\) against the alternative in which \(q^{E_l} = 0\) and the environmental water allocations in all other locations, \(q^E_{-l} \equiv (q^{E_1}, q^{E_2}, ..., q^{E_{l-1}}, q^{E_{l+1}}, ..., q^{EL})\),
are unchanged. Suppose that the utility household \( i \) derives from \( q^E \equiv (q^E_i, q^{\bullet E}) \) is represented by the additive term \( u_i^E(q^E) \), which is added to \( v_i(p_w, p_z, y_i, n_i) \) of (2.4). Household \( i \)'s WTP for \( q^{\bullet E} \) when environmental water allocation is \( q^E \), denoted \( WTP_i^l(q^E) \), is defined by

\[
v_i(p_w, p_z, y_i - WTP_i^l(q^E), n_i) + u_i^E(q^E) = v(p_w, p_z, y_i, n_i) + u_i^E(q^E_{\cdot i}, 0). \tag{2.5}
\]

Estimating households WTP for environmental water belongs to the general area of valuing natural amenities, on which a large (and growing) body of literature exists (see Freeman 2003, Bockstael and McConnell 2007, for recent contributions).

**2.2.5 Consumers (users) surplus**

The gross surplus (not including the water cost) sector \( s \) in location \( l \) derives from consuming the water quantity \( q \) is

\[
B^{sl}(q) = \int_0^q D^{sl}(\alpha) d\alpha, \ s = 1, 2, ..., S, \ l = 1, 2, ..., L. \tag{2.6}
\]

Since \( D^{sl}(\cdot) \) is positive and decreasing, \( B^{sl}(\cdot) \) is increasing and strictly concave. The surplus generated by \( q^{\bullet E} \) is the sum of the \( WTP_i^l(q^E) \) over all households \( i \) in the economy,

\[
B^{Ei}(q^E) \equiv B^{S+1,l}(q^{S+1}) = \sum_i WTP_i^l(q^E), \ l = 1, 2, ..., L
\]

and the surplus generated by \( q^E \) is

\[
B^E(q^E) \equiv B^{S+1}(q^{S+1}) = \sum_{l=1}^L B^{Ei}(q^E). \tag{2.7}
\]

**2.3 Water supply**

Water supply entails extraction-production, conveyance, treatment and distribution. Each activity requires capital, labor, energy and material in-
puts. The capital cost constitutes the bulk of the fixed cost (some labor costs, such as management and accounting, may also be independent of the water supply rate, hence included in the fixed cost), while the costs of the other inputs make up the variable cost. We discuss each in turn.

2.3.1 Capital cost

The capital stock of each activity is measured in terms of the full cost of installing the infrastructure (pipes, pumps, canals etc.) necessary to carry out the activity. The notation used for the various capital stocks is presented in Table 1. A capital stock determines the capacity of the associated supply activity, i.e., the maximal quantity of water that can be supplied during a year, but otherwise has no effect on the water supply rate. We denote these capacity functions by \( F(\cdot) \) with the same subscripts and superscripts as those of the associated capital stock. For example, \( F_{e}^{m}(k) \) is the maximal annual amount of water that can be extracted from source \( m \) when \( K_{e}^{m} = k \).

<table>
<thead>
<tr>
<th>Capital</th>
<th>Capacity</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{e}^{m} )</td>
<td>( F_{e}^{m} )</td>
<td>Extraction from ( m = 1, 2, \ldots, M )</td>
</tr>
<tr>
<td>( K_{des}^{h} \equiv K_{e}^{M+h} )</td>
<td>( F_{des}^{h} \equiv F_{e}^{M+h} )</td>
<td>Desalination plant ( h = 1, 2, \ldots, H )</td>
</tr>
<tr>
<td>( K_{e} )</td>
<td>( F_{e} )</td>
<td>Basin-wide conveyance</td>
</tr>
<tr>
<td>( K_{ml}^{e} )</td>
<td>( F_{ml}^{e} )</td>
<td>Conveyance from ( m \in J^{l} ) to ( l )</td>
</tr>
<tr>
<td>( K_{tr}^{l} )</td>
<td>( F_{tr}^{l} )</td>
<td>Treatment, location ( l )</td>
</tr>
<tr>
<td>( K_{sew}^{s} )</td>
<td>( F_{sew}^{s} )</td>
<td>Sewage, location ( l )</td>
</tr>
<tr>
<td>( K_{rec}^{sl} )</td>
<td>( F_{rec}^{sl} )</td>
<td>Recycling to ( s \in J^{rec} ) in ( l )</td>
</tr>
</tbody>
</table>

Water treatment may occur (i) at the source (upon extraction, before conveyance), (ii) in conjunction with basin-wide conveyance or (iii) upon reaching location \( l \). Regarding (i), at-the-source treatment occurs in conjunction with
extraction and the extraction capital includes in-source treatment capital as well. Likewise, basin-wide treatment is carried out in conjunction with basin-wide conveyance and $K_c$ includes also the treatment capital. Treatment in location $l$ can be carried out centrally for all sectors, using capital $K_{cl}$, or separately for each sector, in which case the distribution capital $K_{sl}$ includes treatment capital as well. Which design is more cost-effective depends on the nature of the location. For example, locations that are predominately urban may prefer central treatment to a drinking quality, whereas locations that are predominately agricultural may prefer separate treatment systems for urban and agricultural users.

Water is conveyed from source $m$ to location $l$ in one of two ways: either directly, using the infrastructure $K_{cm}$ designated solely for that purpose, or via the basin-wide conveyance facility $K_c$. We denote by $J^l$ the set of sources that can supply water directly to location $l$: if $m \in J^l$ then water from $m$ to $l$ is conveyed via $K_{cm}$; if $m \notin J^l$, then water from $m$ to $l$ is conveyed via the basin-wide conveyance facility $K_c$ if location $l$ has access to $K_c$. Notice that $K_{cm}$ can be used only to deliver water from $m$ to $l$. If a conveyance facility serves more than one source-location ($ml$) combination, it is included in $K_c$.\textsuperscript{3} Some locations may not have access to $K_c$ and can receive water only from sources $m \in J^l$. We denote by $J_c$ the set of locations that have access (are connected) to the basin-wide conveyance facility $K_c$.

Sewage activity refers to the mandatory collection and treatment of water from urban and industrial sectors, disregarding whether the treated water will be reused later on. We denote by $J^{sew}$ the set of sectors that are connected to

\textsuperscript{3}In general more than one conveyance systems deliver water to multiple source-location combinations. Here we assume a single $K_c$ system. Allowing for multiple $K_c$ systems will add details but change none of the results.
the sewage system. Typically the sewage infrastructure in location \( l (K_{\text{sew}}^l) \) serves all sectors connected to the sewage system, i.e., all \( s \in J_{\text{sew}} \), hence is not sector-specific (the variable costs of sewage treatment do vary across sectors – see Table 2 below).

Recycling is the voluntary activity of reusing the treated sewage water, which requires further treatment, conveyance and distribution to end-users. Some sectors (e.g., residential) are not allowed to use recycled water and we let \( J_{\text{rec}} \) represent the set of all sectors that can use recycled water. Because recycled water cannot be mixed with drinking water, it requires a distribution system of its own. The recycling infrastructure, \( K_{\text{rec}}^l \), includes treatment, conveyance and distribution facility.

The annual cost of capital is the interest and depreciation on the (current-value) capital stock, which constitutes the bulk of the fixed cost of water supply. For example, with \( r \) and \( \delta \) representing the interest and depreciation rates, respectively, the annual capital cost associated with extraction from source \( m \) is \( (r + \delta)K_c^m \).

### 2.3.2 Variable cost

The variable costs of supply are due to energy, labor and material inputs. They are listed in Table 2. Supplying \( a \) m\(^3\) per year to sector \( s \in J_{\text{sew}} \) in location \( l \) from source \( m \in J_l \) entails the variable cost

\[
\frac{C_{\text{ex}}^m(a)}{\text{extraction}} + \frac{C_{\text{cnt}}^m(a)}{\text{conveyance}} + \frac{C_{\text{tr}}^l(a)}{\text{treatment}} + \frac{C_{\text{d}}^{sl}(a)}{\text{distribution}} + \frac{C_{\text{sew}}^{sl}(a)}{\text{sewage}}.
\]

For \( m \notin J_l \) and \( l \in J_c \), \( C_c \) replaces \( C_{\text{cnt}}^m \), and \( C_{\text{sew}}^{sl} = 0 \) for \( s \notin J_{\text{sew}} \). The variable cost of supplying \( a \) m\(^3\) per year from desalination plant \( h \) to sector \( s \)
Table 2: Variable costs

<table>
<thead>
<tr>
<th>Notation</th>
<th>Variable cost of (activity)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C^m_e$</td>
<td>Extraction (and possibly treating), source $m = 1, 2, ..., M$</td>
</tr>
<tr>
<td>$C_{des} \equiv C^{M+h}_e$</td>
<td>Desalination, plant $h = 1, 2, ..., H$</td>
</tr>
<tr>
<td>$C^m_c$</td>
<td>Conveyance from source $m \in J^c$ to location $l$</td>
</tr>
<tr>
<td>$C_c$</td>
<td>Basin-wide conveyance: relevant for conveyance from $m \notin J^c$ to $l$</td>
</tr>
<tr>
<td>$C^l_{tr}$</td>
<td>Treatment before distribution in location $l$</td>
</tr>
<tr>
<td>$C^l_d$</td>
<td>Distribution (and possibly treatment) in location $l$ to sector $s$</td>
</tr>
<tr>
<td>$C^l_{sl}$</td>
<td>Sewage collection and treatment, sector $s \in J^{sew}$ in location $l$</td>
</tr>
<tr>
<td>$C^l_{rec}$</td>
<td>Recycling: treating, conveying &amp; distributing to sector $s \in J^{rec}$ location $l$</td>
</tr>
</tbody>
</table>

in location $l$ is

$$C_e^{M+h}(a) + C_c^{M+h,l}(a) + C_d^{sl}(a) + C_{sew}^{sl}(a),$$

with the obvious modifications if $M + h \notin J^c$ or $s \notin J^{sew}$. The current state of desalination technology leaves ample room for cost reduction due to technical change (see Tsur and Zemel 2000).

Because mixing recycled with water derived from the other $M + H$ sources is not allowed, recycled water requires conveyance and distribution systems of its own, which are included in the recycled capital $K^{rec}_{rec}$ (Table 1). The variable cost of recycled water supply at the rate $a$ to sector $s$ in location $l$ is $C^{sl}_{rec}(a)$.

### 2.4 Regulator

The regulator, or water authority, oversees and implements the water policy defined next.
3 Water policy

At the beginning of year $t$, after the precipitation $w_t$, hence recharge $x_t$, has been realized, the water state $Z_t = (Q_t, x_t)$ is observed. Given $(Z_t)$, the policy decisions for year $t$ entail: (i) extraction quotas $g^m_t$, $m = 1, 2, ..., M$, for the $M$ naturally replenished sources; (ii) allocation of the extracted and produced (desalinated and recycled) water among the end users; and (iii) investment in the capital infrastructure that determines the capacity of the various supply activities.

The extraction allotments $g^m_t$, $m = 1, 2, ..., M$, should be determined within an intertemporal decision framework that accounts for hydrological considerations associated with sustaining the water sources in the long run given the stochastic nature of precipitation and the ensuing recharge processes. We seek a decision rule that to any feasible realizations of the water state $Z_t$ assigns a feasible extraction quotas (see formulation in Appendix B). The existing literature follows the pioneering work of Burt (1964) and includes the works of Tsur (1990), Tsur and Graham-Tomasi (1991), Provencher and Burt (1994) and Knapp and Olson (1995). This area is still under-explored and the present effort does not change this state-of-affairs. Our focus here is on (ii) and (iii).

3.1 Water allocation

An annual (intratemporal) water allocation is defined in terms of $q^{msl}$: the amount of water to be supplied from source $m$ to sector $s$ in location $l$, $m = 1, 2, ..., M + H + 1$, $s = 1, 2, ..., S + 1$ and $l = 1, 2, ..., L$, where $m = M + H + 1$ represents the recycling source and $s = S + 1$ is the environment sector. An
allocation generates the following sub-aggregate allocations:

\[ q^{m \bullet \bullet} = \sum_{s=1}^{S+1} \sum_{l=1}^{L} q^{msl} \] (extraction-production from \( m \)), \hspace{1cm} (3.1a)

\[ q^{\bullet \bullet l} = \sum_{m=1}^{M+H} \sum_{s=1}^{S+1} q^{msl} \] (allocation to \( l \)), \hspace{1cm} (3.1b)

\[ q^{m \bullet l} = \sum_{s=1}^{S+1} q^{msl} \] (allocation from \( m \) to \( l \)), \hspace{1cm} (3.1c)

\[ q_c = \sum_{l \in J_c} \sum_{m \notin J_l} q^{m \bullet l} \] (basin-wide conveyance), \hspace{1cm} (3.1d)

\[ q^{\bullet s l} = \sum_{m=1}^{M+H} q^{msl} \] (allocation to \( s \) in \( l \)), \hspace{1cm} (3.1e)

\[ q_{sew}^l = \sum_{s \in J_{sew}} \left( q^{\bullet s l} + q^{M+H+1sl} \right) \] (sewage in \( l \)), \hspace{1cm} (3.1f)

and

\[ q_{rec} = \sum_{s \in J_{rec}} \sum_{l=1}^{L} q^{M+H+1sl} \] (total recycled water). \hspace{1cm} (3.1g)

### 3.2 Capital decisions

The analysis pertains to a mature water economy for which the capital infrastructure has already reached some kind of a steady state. By this we mean that the bulk of the infrastructure development has already been carried out and the intra-temporal capital decisions entail replacement of the depreciated capital and possibly some incremental investment to meet a growing demand. The capital decisions entail the investment rates in any of the capital stocks listed in Table 1.

### 3.3 Feasible allocation

An intratemporal water allocation is feasible if all the \( q^{msl} \) are nonnegative, \( q^{M+H+1sl} = 0 \) for \( s \notin J_{rec} \) (exogenous recycled water use restrictions), and the
sub-aggregate allocations satisfy:

\[ q^{m\bullet} \leq g^m, \quad m = 1, 2, \ldots, M \, \text{(extraction quotas)}, \quad (3.2a) \]

\[ q^{m\bullet} \leq F^m_e (K^m_e), \quad m = 1, 2, \ldots, M + H \, \text{(extraction-production capacity)}, \quad (3.2b) \]

\[ q \leq F_c (K_c) \, \text{(basin-wide conveyance capacity)}, \quad (3.2c) \]

\[ q^{m\bullet l} \leq F^{m l}_e (K^{m l}_e) \, \text{for} \, m \in J^l \, \forall l \, \text{(to} \ l \, \text{conveyance capacity)}, \quad (3.2d) \]

\[ q^{\bullet l} \leq F^{l}_t (K^l_t) \, \forall l \, \text{(treatment in} \ l \, \text{capacity)}, \quad (3.2e) \]

\[ q^{\bullet l} \leq F^{l}_d (K^{l d}_d) \, \forall (l, s) \, \text{(distribution to} \ s \, \text{in} \ l \, \text{capacity)}, \quad (3.2f) \]

\[ q^l_{\text{sew}} \leq F^{l}_{\text{sew}} (K^l_{\text{sew}}) \, \forall l \, \text{(sewage in} \ l \, \text{capacity)}, \quad (3.2g) \]

\[ q^{M+H+1} l \leq F^{s l}_{\text{rec}} (K^{s l}_{\text{rec}}), \quad s \in J^{\text{rec}}, \forall l \, \text{(recycled to} \ sl \, \text{capacity)} \quad (3.2h) \]

and

\[ q_{\text{rec}} \leq (1 - \alpha_{\text{rec}}) \sum_{l=1}^{L} q^l_{\text{sew}} \, \text{(total recycling)}, \quad (3.2i) \]

where \( \alpha_{\text{rec}} \) is the fraction of water loss due to sewage treatment and recycling.

Feasible capital investments are nonnegative and cannot exceed some exogenous bounds (affordable expenditures):

\[ K_t - K_{t-1} (1 - \delta) \geq 0 \, \text{(irreversible capital)}, \quad (3.2j) \]

and

\[ K_t - K_{t-1} (1 - \delta) \leq \bar{I} \, \text{(affordable investment)}, \quad (3.2k) \]

where \( \delta \) is the depreciation rate and \( \bar{I} \) is the exogenous upper bound on investment. Constraints (3.2j)-(3.2k) apply to each capital stock in Table 1.

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3.4 Optimal allocation

An allocation generates the annual surplus

$$S = \sum_{s=1}^{S} \sum_{l=1}^{L} B^s(q^{sl}) + \sum_{l=1}^{L} \sum_{s\in J^{rec}} B^l(q^{M+H+1sl}) + B^E(q^E)$$  \hspace{1cm} (3.3)$$

and inflicts the variable cost

$$\sum_{m=1}^{M+H} C^m_e(q^{m\bullet}) + \sum_{m\in J^l} C^ml_e(q^{m\bullet}) + C^c_e(q^c) + \sum_{l=1}^{L} C^l_{tr}(q^{\bullet l})$$

$$+ \sum_{s=1}^{S+1} \sum_{l=1}^{L} C^sl_d(q^{\bullet sl}) + \sum_{s\in J^{sew}} \sum_{l=1}^{L} C^sl_{sew}(q^{\bullet sl}) + \sum_{s\in J^{rec}} \sum_{l=1}^{L} C^sl_{rec}(q^{M+H+1sl})$$  \hspace{1cm} (3.4)$$

and the capital cost (the interest and depreciation on the aggregate capital stock)

$$(r + \delta) \left\{ \sum_{m=1}^{M+H} K^m_e + \sum_{l=1}^{L} \sum_{m\in J^l} K^ml_e + K^c_e + \sum_{l=1}^{L} K^l_{tr} + \sum_{l=1}^{L} \sum_{s=1}^{S+1} K^sl_d \\
+ \sum_{l=1}^{L} K^l_{sew} + \sum_{s\in J^{rec}} \sum_{l=1}^{L} K^sl_{rec} \right\}.$$  \hspace{1cm} (3.5)$$

Net annual benefit equals the aggregate surplus minus the variable cost minus the capital cost. The optimal allocation is the feasible allocation that maximizes the net annual benefit.

The capital cost (3.5) ought to be explained. Recall that we consider a mature water economy – one in which the capital infrastructure has already reached a steady state (with a possible growth trend). Therefore, the cost of a capital stock $K$ (which represents the full cost of installing the infrastructure) consists of the cost of financing $K$, i.e. the interest payment $rK$, plus the replacement cost $\delta K$ due to depreciation.
4 Optimal pricing

We characterize the water prices that implement the optimal allocation for the private sectors $s = 1, 2, ..., S$, assuming the environmental allocations $q^{mS+1} = q^{mEl}$, $m = 1, 2, ..., M + H + 1$, $l = 1, 2, ..., L$, are given.\(^4\) Derivations and technical details are presented in Appendix A. Sector $s$ in location $l$ constitutes an end user, called user $sl$. There are $S \times L$ such users. The water price user $sl$ faces is specified in terms of intermediate prices associated with extraction and desalination, conveyance, distribution-treatment in each location and sewage collection-treatment. We discuss each in turn.

4.1 Extraction-production

The extracting firms pay (the regulator) an abstraction fee for each water unit ($m^3$) pumped from a naturally-replenished source. This charge, denoted $\Delta^m$, varies across the $M$ sources and represents the scarcity of water at that source. $\Delta^m = 0$ if $g^m \geq F^m(K^m)$, i.e., if the extraction quota is not binding; otherwise it is determined such that extraction from source $m = 1, 2, ..., M$ does not exceed the quota $g^m$. No scarcity rent is imposed on desalination (for all practical purposes, the sea is an unlimited water source), so $\Delta^{M+h} = 0$ for $h = 1, 2, ..., H$.

After extraction and in-source treatment the water price is

$$p^m_e = \Delta^m + c^m_e + \frac{r + \delta}{f^m_e}, \quad m = 1, 2, ..., M + H,$$

(4.1)

where $c^m_e \equiv C^m_e(q^{m**})$ is the marginal cost of extraction (production) from source $m$ and $f^m_e \equiv F^m_e(K^m_e)$ is the marginal product of extraction (production) capital at source $m$, i.e., the increase in the extraction capacity associated

\(^4\)Due to the public good nature of environmental water, its allocation cannot use pricing and will not be further discussed here.
with a marginal (unit) increase in the extraction capital (all derivatives are evaluated at the optimal water and capital allocation).

The \((r + \delta)/f^m_e\) term in equation (4.1) is the marginal cost of extraction (production) capital per unit water. To see this note that, when source \(m\)'s extraction capacity constraint is binding, \(f^m_e\) is the increase in water extraction associated with a marginal (unit) increment in the extraction capital \(K^m_e\). Thus, \(1/f^m_e\) is the incremental capital per unit water, which when multiplied by \((r + \delta)\) gives the annual cost of the incremental capital per unit water.

### 4.2 Conveyance

The intermediate conveyance price is the marginal cost of conveying water from source \(m\) to location \(l\):

\[
p^{ml}_c = \begin{cases} 
  c_c + \frac{r + \delta}{f_c(K_c)} & \text{if } m \notin J^l \text{ and } l \in J_c \\
  c^{ml}_c + \frac{r + \delta}{f^{ml}_c(K^{ml}_c)} & \text{if } m \in J^l 
\end{cases}, \quad l = 1, 2, ..., L, \ m = 1, 2, ..., M+H
\]

(4.2)

where \(c_c \equiv C'_c(q_c)\), \(c^{ml}_c \equiv C^{ml}_c(q^{ml}_c)\), \(f_c \equiv F'_c(K_c)\) and \(f^{ml}_c \equiv F^{ml}_c(K^{ml}_c)\) (all derivatives evaluated at the optimal allocation). Note that if \(m \notin J^l\) (i.e., no facility is solely designated to convey water from \(m\) to \(l\)) and \(l \notin J_c\) (i.e., \(l\) has no access to the basin-wide conveyance facility), then it is impossible to convey water from \(m\) to \(l\) and \(p^{ml}_c\) does not exist.

### 4.3 Treatment and distribution in location \(l\)

Upon reaching location \(l\) the water is treated and distributed to the various sectors. The marginal cost of this operation is

\[
p^{sl}_d = c^{sl}_d + \frac{r + \delta}{f^{sl}_d} + c_{tr} + \frac{r + \delta}{f_{tr}}
\]

(4.3)
where $c_{dl}^s \equiv C_{dl}^s(q^{*sl})$, $f_{dl}^s \equiv F_{dl}^s(K_{dl}^s)$, $c_{tr}^l = C_{tr}^l(q^{*sl})$ and $f_{tr}^l \equiv F_{tr}^l(K_{tr}^l)$ (all derivatives evaluated at the optimal allocation). The first and second terms on the right-hand side of (4.3) represent the marginal cost of distribution to sector $s$ in location $l$ and may include also treatment costs if water is treated separately for sector $s$. The third and fourth terms represent cost of treatment before the water enters the distribution system. In locations that do not perform central treatment, the last two terms vanish.

4.4 Sewage

The prices considered so far are associated with supplying water from the various sources to end-users. The sewage of some sectors, i.e., $s \in J_{sew}$ (urban and industrial sectors), must be collected and treated. The marginal cost of this operation is ($\$/\text{per} \ m^3$)

$$p_{sl}^{sew} = c_{sew}^{sl} + \frac{r + \delta}{f_{sew}^l} \text{ for } s \in J_{sew} \text{ and } \forall l,$$

$p_{sew}^{sl} = 0 \text{ for } s \notin J_{sew}$, where $c_{sew}^{sl} \equiv C_{sew}^{sl}(q^{*sl})$ and $f_{sew}^l \equiv F_{sew}^l(K_{sew}^l)$.

4.5 Recycling

Recycling occurs when the treated sewage water is delivered to user $sl$, which often entails further treatment to the quality required by the receiving sector. The marginal cost of recycling is

$$p_{rec}^{sl} = c_{rec}^{sl} + \frac{r + \delta}{f_{rec}^l} \text{ for } s \in J_{rec} \text{ and } \forall l,$$

where $c_{rec}^{sl} \equiv C_{rec}^{sl}(q^{M+H+1sl})$ and $f_{rec}^l \equiv F_{rec}^l(K_{rec}^l)$.

4.6 End-user prices

We turn now to formulate the optimal end-user prices. To that end, let $I^{sl}$ be the set of all water sources aside from recycling for which $q^{msl} > 0$ under
the optimal allocation:

$$I^{sl} = \{m \in \{1, 2, ..., M + H\} | q^{msl} > 0\}. \quad (4.6)$$

It is easy to detect the exclusion of a particular source from $I^{sl}$. Let

$$\hat{p}^{sl} = D^{sl}(0), \quad s = 1, 2, ..., S, \quad l = 1, 2, ..., L, \quad (4.7)$$

represent the maximal water price below which sector $s$ in location $l$ (i.e., user $sl$) demands a positive amount of water (this is the price user $sl$ will pay for the first water unit). Then, $q^{msl} = 0$ when the water price of source $m$ is equal to or exceeds $\hat{p}^{sl}$, implying that $m \notin I^{sl}$. The $\hat{p}^{sl}$ of the urban sectors are much higher than those of the agricultural sectors, and those of the industrial sectors are typically in between.

The $I^{sl}$ sets of some urban sectors contain all sources (otherwise, the excluded sources will never be demanded and should not be included in the list of water sources), while those of the agricultural sectors typically contain subsets of the $M + H$ sources, e.g., the desalination sources will be excluded from the $I^{sl}$ of most agricultural sectors in most locations. Let $M^{sl}$ indicate the number of sources included in $I^{sl}$, so $M^{sl} \leq M + H$ with equality holding for at least one end-user $sl$.

Let

$$\bar{p}^{sl} = \frac{1}{M^{sl}} \sum_{m \in I^{sl}} (p^m_e + p^m_l) \quad (4.8)$$

represent the average marginal cost of supplying water to user $sl$, averaged over the $M + H$ sources (excluding recycling) from which user $sl$ demands water (i.e., over the sources included in $I^{sl}$). We are now ready to state the main result:
Property: The optimal $S \times L$ end-user prices of water derived from sources $m = 1, 2, ..., M + H$ are

$$p^{sl} = \bar{p}^{sl} + p_d^{sl} + p_{sew}^{sl}, \ s = 1..., S, \ l = 1,..., L. \quad (4.9)$$

The property implies that the end user prices $p^{sl}$ are not directly affected by the cost of water derived from sources that are irrelevant to user $s_l$, i.e., excluded from the $I^{sl}$ set. For example, the cost of desalination should not directly affect the price of irrigation water in agriculture sectors for which $\hat{p}^{sl} \leq p_c^{M+h} + p_c^{M+h'}, \ h = 1, 2, ..., H$ (which is the case in Israel for all agricultural sectors). However, the desalination price will affect the price of irrigation water indirectly via its affect on water scarcity. A higher desalination cost reduces the scale of desalination, thereby increasing the scarcity prices, $\Delta^m$, $m = 1, 2, ..., M$, of the natural water sources.

As recycled water ($m = M+H+1$) uses separate treatment and conveyance facilities, it is priced separately of water derived from the other $M+H$ sources. The end-user prices of recycled water are

$$p_{rec}^{sl} + p_{sew}^{sl}, \text{ for } s \in J^{rec}, \quad (4.10)$$

where $p_{rec}^{sl}$ and $p_{sew}^{sl}$ are defined in (4.5) and (4.4), respectively.

4.7 Supply stages and intermediate prices

The supply process can be viewed as proceeding along the following stages:

The extracting firms are restricted not to exceed the extraction allotments $g_t = (g_t^2, g_t^2, ..., g_t^M)$, determined by the regulator. Alternatively, the regulator can charge the extraction fees $\Delta^m$, $m = 1, 2, ..., M$, determined such that the extraction firms will not extract beyond the extraction quotas. The extracted
(produced) water is “sold” to the conveyance firms at the price \( p^m_e \). The conveyance firms deliver the water to the \( L \) locations, charging location \( l \) the price

\[
p^l_c = \frac{1}{q^{sl}} \sum_{s=1}^{S} \bar{p}^{sl} q^{sl}, \quad l = 1, 2, ..., L, \quad (4.11)
\]

where \( \bar{p}^{sl}, q^{sl} \) and \( q^{sl} \) are defined in (4.8), (3.1b) and (3.1e), respectively.\(^5\)

Location \( l \)'s water authority, then, treats and distributes the water to end users in its location, and collects and treats the sewage, charging end-users the price \( p^l \). We summarize the intermediate prices associated with each stage in Table 3.

<table>
<thead>
<tr>
<th>Price</th>
<th>Received by</th>
<th>Payed by</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta^m )</td>
<td>Regulator</td>
<td>Source ( m )'s extraction firm</td>
</tr>
<tr>
<td>( p^m_m )</td>
<td>Source ( m )'s extraction firm</td>
<td>Conveyance firms</td>
</tr>
<tr>
<td>( p^l_c )</td>
<td>Conveyance firms</td>
<td>Location ( l )'s water authority</td>
</tr>
<tr>
<td>( p^l )</td>
<td>Location ( l )'s water authority</td>
<td>User ( sl )</td>
</tr>
</tbody>
</table>

### 5 Regulation

The regulator’s task at the beginning of year \( t \) entails (a) ensuring that extractions from the \( M \) natural sources do not exceed \( g_t = (g^1_t, g^2_t, ..., g^M_t) \) (see Appendix B on the optimal \( g_t \)) and (b) allocating the overall extracted and produced water among the end users. The policy tools available to the regulator are prices and quotas. The pros and cons of prices vs. quotas have

\(^5\)The water proceeds of location \( l \)'s water authority are \( \sum_s \bar{p}^{sl} q^{sl} + \sum_s (p^d_s + p^l_{sewl}) q^{sl} \). The second sum is used to cover cost of treatment, distribution and sewage collection in the location. The first sum is used to “buy” the water quantity \( q^{sl} \) from the conveyance firms, which is the same as buying that quantity at the price \( p^l_c \).
been shown by Weitzman (1974) to depend on the balance between (i) the underlying uncertainty or asymmetric information and (ii) the elasticity of demand and supply. A thorough investigation of this issue in the context of water regulation is beyond the present scope.

At the beginning of period $t$ (after the precipitation has been realized), the regulator knows (calculates) $g_t$ (see Appendix B) and can accomplish task (a) by setting the quotas $g_t$ on extractions from the $M$ natural source. Alternatively, the regulator can set the extraction charges (also known as scarcity rents or user cost) $\Delta^m$ such that the extracting firms will find it undesirable to extract beyond the $g^m$ quotas. Setting these charges appropriately (not too high and not too low) requires knowledge of the water demand supply relations, on which the regulator rarely has full information. Regulating extractions solely by the extraction charges $\Delta^m$ is therefore not recommended. A third alternative is a combination of quotas and low extraction charges: the quotas ensure that extractions do not exceed the limits $g_t$ and the proceeds from the extraction charges can be used to cover various regulation expenses.

The remaining of this section deals with regulation task (b).

5.1 Allocation regulation

The optimal prices defined above are evaluated at the optimal allocation, where end-user demands and supply costs intersect. Calculating these prices in actual practice requires information on the demands of all end-users and all supply costs. This information is rarely available to water authorities. The regulation task, it turns out, is greatly simplified under the special case of constant returns to scale supply technologies. We begin with this special case.
5.1.1 Linear prices

Suppose that the capacity and variable cost functions listed in Tables 1 and 2 are of the form \( C(a) = ca \) and \( F(K) = fK \). Thus, the marginal cost \( c \) equals the average cost independent of the supply rate, and the marginal capacity \( f \) equals water supply per unit capital independent of the capital stock. In this case, the intermediate and end-user prices are independent of the supply rates and can be determined without recourse to water demands nor to the optimal allocation. Moreover, the water proceeds exactly cover the full cost (variable and fixed) of water supply.

How does the regulator find out the true average costs, i.e., the \( c \)'s and \( f \)'s of the various \( C(\cdot) \) and \( F(\cdot) \) functions? Often the information available to the regulator comes from activity reports (e.g., balance sheets) of the water supply firms, giving rise to myriad of agency problems. The literature offers a variety of methods to overcome or mitigate such problems (see Laffont and Tirole 1986, 1993, for relevant contributions). For example, by setting a price cap based on observed (reported) average costs with a period of gradual reduction to a target (lower) price. Firms that outperform the curve (i.e., become efficient faster) can keep the extra profits, while firms that trail the curve will be replaced. When feasible, auctions should be used to choose the operating firms. For example, the choice of a desalination firm, or the firm to build and operate an irrigation project.

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6The firms, knowing that their reported information may be used against them (i.e., to determine efficient prices) are likely to misrepresent true costs.
5.1.2 Linear prices as second-best regulation

The pervasiveness of scale economies in water supply technologies renders unlikely the linearity of the cost and capacity functions. In such cases the average costs differ from the marginal costs and both vary with the water allocation. The task of calculating the optimal prices, then, requires information on the water demands of all end users and the supply costs of all supply firms and quickly becomes intractable. Moreover, aside from the information issue, under the optimal, marginal cost prices the water proceeds do not cover the full cost of supply. Imposing the constraint that the water proceeds cover the supply cost, then, implies departure from the optimal, marginal cost pricing rule. The Ramsey rule (Ramsey 1927) specifies a departure that maximizes aggregate consumer surpluses subject to balanced supply budgets (see, e.g., Wilson 1993, Chapter 5). This rule requires information on the demand elasticities of all sectors. Lacking this information, the regulator may resort to a simple average cost pricing, by setting the $c$’s and $f$’s of the various prices at the associated average costs. This simple average cost pricing rule balances the supply budgets but is suboptimal to the Ramsey pricing rule. Given the information limitation, it is viewed as second-best pricing.

5.1.3 Decentralized regulation

The pricing problems discussed above stem from the so-called asymmetric information – when consumers and suppliers have private information that they may not disclose (see Smith and Tsur 1997, Tsur 2000, for water related discussions). Decentralization, namely delegating decisions to consumers and suppliers, is often an effective way to overcome or mitigate such problems. Water markets are (extreme) examples of decentralized mechanisms. Trad-
ing can be in water, in water rights or in water quotas, it may be formal or informal and it can be carried out within and between sectors (e.g., irrigation associations and urban districts) as well as within and between time periods. The wide range of observed market designs stem from the wide range of institutional, hydrological and physical setting affecting the operation of water markets (see Easter et al. 1998, 1999, Dudley 1999, Zilberman and Schoengold 2005, 2007, and references they cite). They all serve to alleviate problems associated with asymmetric information.

6 Concluding remarks

The above is a bare-bones account of basic principles of water regulation. In actual practice one encounters a myriad of problems that lead to departure from these basic principles and require special considerations. The asymmetric information problem has been discussed in Section 5. We remark on a few additional, frequently encountered, issues.

Subsistence water Water for basic needs (drinking, cooking, hygiene) is considered by many as a human right to which all are entitled, disregarding economic considerations such as supply costs and households’ budget constraints. The manifestation of this view in actual practice is via block-rate pricing of urban water, with a low (or even zero) price for the subsistence block (see Gleick 1996, for basic water needs).

Implementation costs The prices formulated above are volumetric and require metered water or some other way to infer the volume of water consumed. Volumetric pricing entails implementation costs, associated with installing and
maintaining water meters, monitoring water use and collecting fees. These costs are high relative to other pricing methods (which may explain why worldwide the bulk of irrigation water is unmetered—see Bos and Wolters 1990). When implementation costs are included in the welfare calculations, other pricing methods, such as area pricing, may outperform volumetric prices (see Tsur and Dinar 1997, for some examples in the case of irrigation water).

**Water laws and institutions** Water laws, ownership rights and water institutions determine the toolkit available for policymaking and often limit the use of prices and quotas in implementing a water policy (Rausser and Zusman 1991, Zusman 1997, Saleth and Dinar 2004, Griffin 2006, among many others). These constraints, when added to the feasibility constraints imposed on the water allocation problem, could lead to substantially different policies.

**Nonstationary demand** Water demands increase in time due to demographic and economic growth. On the supply side, the recharge processes of the natural sources $m = 1, 2, \ldots, M$, although fluctuating from year to year, are stationary (up to possible long-run trends associated with climate change). Driven by the hydrological base and the stationary recharge processes, the extraction quotas $g_t = (g^1_t, g^2_t, \ldots, g^M_t)$ from the $M$ natural sources cannot grow beyond certain limits. Eventually, the growing demand will have to be met by produced (desalinated and recycled) water. Water-abundant or sparsely-populated regions do not need the produced sources (at least not in the near term). But many water-scarce or densely-populated regions already need these sources and the number of such regions increases every year.
Private sectors with public good features  Water used in some private sectors may also have public-good effects. Examples include landscape amenities of irrigated farmland and private urban gardens (McConnell 1989, Drake 1992). Fleischer and Tsur (2009) showed that the landscape amenity of a particular (irrigated) agricultural sector $s$ (a crop or a group of crops) increases the value of marginal product of land, hence also of water, for this sector. In such cases, the social water demand (that accounts for the external landscape effects) lies above the private water demand $D^{sl}(\cdot)$, defined in subsection 2.2.1. The optimal water prices for this sector should be determined according to the social demand schedule rather than the private schedule $D^{sl}(\cdot)$ and the ensuing optimal allocation entails more water to sector $s$ compared with the allocation based on the private demand $D^{sl}(\cdot)$. Such effects may justify subsidizing irrigation water for certain agricultural sectors, e.g., by setting a lower price up to a certain quantity of water (i.e., a form of block-rate pricing).

General equilibrium considerations  Our analysis is of a partial equilibrium type in that we assume that the rest of the economy is exogenous to the water economy. For example, we take perimetrically the price of capital (the interest rate $r$). Often, the water economy constitutes a substantial part of the entire economy, to the extent that the water policy may have feedback effects with a number of economy-wide variables, such as the price of capital and labor. In such cases economy-wide considerations can have significant ramifications on water regulation (see e.g. Tsur et al. 2004, Diao et al. 2008).
Appendix

A Derivation of the optimal prices

Environmental water allocations are assumed exogenous and set at zero for convenience. We seek the water allocation \( \{q_{msl}\} \) and the capital allocation \( \{K_{me}, K_{mlc}, K_{c}, K_{ltr}, K_{slsd}, K_{lssew}\} \), \( m = 1, 2, ..., M + H + 1 \), \( s = 1, 2, ..., S \), \( l = 1, 2, ..., L \), that maximize

\[
\sum_{l=1}^{L} \sum_{s=1}^{S} B_{sl}(q_{sl}^{*}) + \sum_{l=1}^{L} \sum_{m \in J_{rec}} B_{sl}(q_{M+H+1sl}^{*}) - \left\{ \sum_{m=1}^{M+H} C_{c}^{m}(q_{m}^{**}) + \sum_{l=1}^{L} \sum_{m \in J} C_{c}^{ml}(q_{m}^{**}) + C_{c}(q_{c}) + \sum_{l=1}^{L} C_{tr}^{l}(q_{l}^{**}) \right\} \\
+ \sum_{s=1}^{S} \sum_{l=1}^{L} C_{sd}^{sl}(q_{sl}^{*}) + \sum_{l=1}^{L} \sum_{s \in J_{sew}} C_{sew}^{sl}(q_{sl}^{*}) + \sum_{l=1}^{L} \sum_{s \in J_{rec}} C_{rec}^{sl}(q_{M+H+1sl}^{*}) \right\} \\
-(r + \delta) \left\{ \sum_{m=1}^{M+H} K_{me}^{m} + \sum_{l=1}^{L} \sum_{m \in J} K_{c}^{ml} + K_{c} + \sum_{l=1}^{L} K_{ltr}^{l} + \sum_{l=1}^{L} \sum_{s=1}^{S} K_{sl}^{sl} + \sum_{l=1}^{L} \sum_{s \in J_{rec}} K_{sl}^{sl} \right\} \tag{A.1}
\]

subject to the feasibility constraints (3.2), exogenous constraints regarding water quality (affecting treatment requirement and recycled water allocation) and nonnegativity of the water allocations, given the previous year capital stocks (the sub-aggregate allocations are specified in (3.1)).

Notice that, given the previous year capital stocks, the capital decisions entail only this year investments. Notice also that it cannot be optimal to plan idle capacity in any of the capital stocks (since it increases the cost without any benefit compensations). In actual practice, the extraction allotments \( g_t = (g^1_t, g^2_t, ..., g^M_t) \) vary from year to year, based on the precipitation realization.
and the extraction capital stocks $K_m^e$, $m = 1, 2, ..., M$, are set according to some average allotment vector $\bar{g}$ and the other capital stocks are determined accordingly (with no idle capital).\textsuperscript{7} We solve (A.1) for an average year in which $g_t = \bar{g}$, so (3.2b) are binding and (3.2a) represents the same constraints as (3.2b), hence can be ignored. We also assume that (3.2j)-(3.2k) are non-binding.

We use the following notation:

\begin{align*}
  c_{ml}^c &= \begin{cases}
    C'_c(q_m) & \text{if } m \notin J_l \text{ and } l \in J_c \\
    C^{ml}_c(q_m^*) & \text{if } m \in J_l
  \end{cases} \\
  c_{sl}^{sew} &= \begin{cases}
    C_{sew}^s(q^{\bullet s}) & \text{if } s \in J_{sew} \\
    0 & \text{otherwise}
  \end{cases} \\
  c_{sl}^{rec} &= \begin{cases}
    C_{rec}^s(q^{M+H+1s}) & \text{if } s \in J_{rec} \\
    0 & \text{otherwise}
  \end{cases}
\end{align*}

\textsuperscript{7}The optimal $\bar{g}$ according to which the extraction capital stocks are determined must be specified within an intertemporal decision problem and will not be pursued here.

In general, lower-case $c(\cdot)$ indicates the marginal cost (derivative) of the corresponding cost function $C(\cdot)$ and lower-case $f(\cdot)$ stands for the marginal product (derivative) of the corresponding capacity function $F(\cdot)$. $\mu^m_e$ is the shadow price of (3.2b), $\mu^{ml}_e$ is the shadow price of (3.2c) or (3.2d) for \{\(m \notin J_l \text{ and } l \in J_c\) or \{\(m \in J_l \text{ and } \forall l\), respectively; $\mu^{tr}_l$ is the shadow price of (3.2e); $\mu^{sl}_d$ is the shadow price of (3.2f); $\mu^{sew}_s$ is the shadow price of (3.2g); and $\mu^{rec}_s$, $s \in J_{rec}$, is the shadow price of (3.2h). We assume that (3.2i) is not binding.
Necessary conditions for optimum include:

\[
D^{sl}(q^{\bullet sl}) - c^{sl}_{e}(q^{\bullet m}) - c^{ml}_{c} - c^{d}_{tr}(q^{\bullet sl}) - c^{d}_{sew} - \\
(\mu^{m}_{c} + \mu^{ml}_{c} + \mu^{d}_{tr} + \mu^{sl}_{d} + \mu^{sl}_{sew}) \leq 0, \ m = 1, 2, ..., M + H, \ \forall(s,l) \tag{A.5}
\]
equality holding if \(q^{msl} > 0\), where \(\mu^{sl}_{sew} = \mu^{l}_{sl}_{sew} = 0\) for \(s \in J^{sew}\) or \(s \notin J^{sew}\), respectively; for \(m = M + H + 1\) (recycled water)

\[
D^{sl}(q^{M+H+1sl}) - c^{sl}_{sew} - c^{sl}_{rec} - \mu^{sl}_{sew} - \mu^{sl}_{rec} \leq 0, \ s \in J^{rec}, \tag{A.6}
\]
equality holding if \(q^{M+H+1sl} > 0\);

\[
\mu^{m}_{e} = \frac{r + \delta}{f^{m}_{e}(K^{m}_{e})} \tag{A.7}
\]
if (3.2b) is binding, \(\mu^{m}_{e} = 0\) otherwise;

\[
\mu^{ml}_{c} = \begin{cases} 
\frac{r + \delta}{f^{l}_{l}(K^{l}_{c})} & \text{if } \ m \notin J^{l}, \ l \in J^{c} \text{ and (3.2c) is binding} \\
\frac{r + \delta}{f^{m}_{e}(K^{m}_{c})} & \text{if } \ m \in J^{l} \text{ and (3.2d) is binding}
\end{cases} \tag{A.8}
\]

\(\mu^{ml}_{c} = 0\) if (3.2c) or (3.2d) are not binding (recall that if \(m \notin J^{l}\) and \(l \notin J^{c}\) then no water can be delivered from \(m\) to \(l\) and \(\mu^{ml}_{c}\) does not exist);

\[
\mu^{d}_{tr} = \frac{r + \delta}{f^{d}_{tr}(K^{d}_{tr})} \tag{A.9}
\]
if (3.2e) is binding, \(\mu^{d}_{tr} = 0\) otherwise;

\[
\mu^{sl}_{d} = \frac{r + \delta}{f^{sl}_{d}(K^{sl}_{d})} \tag{A.10}
\]
if (3.2f) is binding, \(\mu^{sl}_{d} = 0\) otherwise;

\[
\mu^{sl}_{sew} = \begin{cases} 
\frac{r + \delta}{f^{sl}_{sew}(K^{sl}_{sew})} & \text{if } s \in J^{sew} \text{ and (3.2g) is binding} \\
0 & \text{otherwise}
\end{cases} \tag{A.11}
\]
\[
\mu^{sl}_{rec} = \begin{cases} 
\frac{r + \delta}{f^{sl}_{rec}(K^{sl}_{rec})} & \text{if } s \in J^{rec} \text{ and (3.2h) is binding} \\
0 & \text{otherwise}
\end{cases} \tag{A.12}
\]
No slack capital under the optimal allocation implies binding capacity constraints and we can define (all functions are evaluated at the optimal allocation):

$$p^m_e \equiv c^m_e + \mu^m_e + \lambda^m = c^m_e + (r + \delta)/f^m_e + \Delta^m,$$

(A.13a)
giving rise to (4.1),

$$p^{ml}_e \equiv c^{ml}_e + \mu^{ml}_e = \begin{cases} c^e + \frac{r + \delta}{f^m_e(K^m_e)} & \text{if } m \notin J^l \text{ and } l \in J^c, \\ c^{ml}_e + \frac{r + \delta}{f^{ml}(K^{ml}_{ml})} & \text{if } m \in J^l, \end{cases},$$

(A.13b)
as specified in (4.2),

$$p^{sl}_d \equiv c^l_{tr} + c^{sl}_d + \mu^l_{tr} + \mu^{sl}_d = c^l_{tr} + c^{sl}_d + \frac{r + \delta}{f^{sl}_{tr}} + \frac{r + \delta}{f^{sl}_{d}},$$

(A.13c)
as in (4.3),

$$p^{sl}_{sew} \equiv c^{sl}_{sew} + \mu^{sl}_{sew} = \begin{cases} c^{sl}_{sew} + \frac{r + \delta}{f^{sl}_{sew}(K^{sl}_{sew})}, & s \in J^{sew}, \\ 0 & \text{otherwise} \end{cases},$$

(A.13d)
as in (4.4), and

$$p^{sl}_{rec} \equiv c^{sl}_{rec} + \mu^{sl}_{rec} = c^{sl}_{rec} + \frac{r + \delta}{f^{sl}_{rec}(K^{sl}_{rec})} \quad \text{for } s \in J^{rec},$$

(A.13e)
as in (4.5).

With $I^{sl}$ as the set of all water sources $m$ for which $q^{msl} > 0$ under the optimal allocation, (A.5) holds as equality for all $m \in I^{sl}$. Summing (A.5) over all $m \in I^{sl}$ and dividing by $M^{sl}$ (the number of sources in $I^{sl}$) gives

$$D^{sl} = \bar{p}^{sl} + p^{sl}_d + p^{sl}_{sew},$$

(A.14)
where $\bar{p}^{sl}$ is defined in (4.8). Noting that, evaluated at the optimal allocation, $D^{sl}$ is the optimal water price for sector $s$ in location $l$, verifies 4.9.

Noting (A.6), if $q^{M+H+1} > 0$

$$D^{sl}(q^{M+H+1sl}) = c^{sl}_{rec}(q^{M+H+1sl}) + c^{sl}_{sew} + \mu^{sl}_{rec} + \mu^{sl}_{sew} = p^{sl}_d + p^{sl}_{sew}, \quad s \in J^{rec},$$

verifying (4.10), noting that the left-hand side is the demand price when a positive amount of recycled water is consumed.
The water state at period $t$ is represented by
\[ Z_t \equiv (Q_t, x_t), \tag{B.1} \]
where $Q_t$ and $x_t$ are, respectively, the water stocks and recharge, defined in (2.2) and (2.3). Let $f(Z'\mid Z, g)$ denote the state’s transition density, i.e., the pdf of $Z_{t+1}$, conditional on $Z_t = Z$ and $g_t = g$, evaluated at $Z' = (Q', x')$. From (2.2)-(2.3) we obtain
\[ f(Z'\mid Z, g) = f_w \Lambda (Q' - Q - R(Q) + g - x \Gamma) I(x' = w \Lambda + x \Gamma) \tag{B.2} \]
where $I(\cdot)$ is the indicator function that assumes the values 1 when its argument is true and 0 otherwise, and $f_w(\cdot)$ is the pdf of $w$ induced by $F_w$.

Denote by $B(Z_t, g_t)$ the annual net benefit at year $t$ characterized above, where $g_t$ is the vector of $M$ extraction quotas during year $t$ and $Z_t$ is the state vector, defined in (B.1). Given the initial state $Z_0 = (Q_0, x_0)$, the precipitation series $\{w_t\}_{t=1,2,...}$ generates $\{x_t\}_{t=1,2,...}$ via (2.2), which together with the policy $\{g_t\}_{t=1,2,...}$ generates $\{Q_t\}_{t=1,2,...}$ via (2.3), giving rise to the (random) payoff
\[ \sum_{t=1}^{\infty} \beta^t B(Z_t, g_t), \]
where $\beta \in (0, 1)$ is a constant discount factor.

Given $Z_0 = Z$, the value function, $v(Z)$, is the maximal expected payoff over all feasible extraction policies $\{g_t\}_{t=1,2,...}$,
\[ v(Z) = \max_{\{g_t\}} \sum_{t=1}^{\infty} E_t \left\{ \beta^t B(Z_t, g_t) \right\}, \tag{B.3} \]
where $E_t$ signifies expectation conditional on information available at time $t$ (i.e., $Z_{t-1}$). Then, $v(\cdot)$ satisfies the optimality equation
\[ v(Z) = \max_{g \in A(Z)} \left\{ B(Z, g) + \beta \int v(Z') f(Z'\mid Z, g) dZ' \right\}, \tag{B.4} \]

where \( A(Z) \) is the set of feasible extractions at water state \( Z \) and \( f \) is the transition density defined in (B.2).

A Markov extraction policy \( g(Z) \) is a rule assigning a feasible \( g \) to any feasible state \( Z \). The optimal policy \( g^*(Z) \) is the extraction rule that maximizes the right-hand side of (B.4). An important line of research entails studying the properties of the optimal extraction policy, such as existence and uniqueness of \( g^*(\cdot) \) as well as convergence of the optimal state process to a steady state distribution under various recharge processes, water demand forms and supply technologies (Puterman 2005, is a good resource for this task).

References


